MARITIME UNIVERSITY OF SZCZECIN

Institute of Marine Traffic Engineering

Instruction No. 00

Formulas for statistical calculations in radionavigation exercises

Prepared by::	Renata Boć
Approved by:	Stefan Jankowski
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Topic: Statistical formulas for calculations in exercises No 6- 10

1. Determination of the average position error for the given geographical coordinates φ and λ.

On the basis of the measurement results obtained, in order to φ_1, φ_2, … ,φ_n and λ_1, λ_2, … , λ_n calculate the mean square errors of the measurements according to the following calculation scheme (assuming measurements from a stationary receiver - the position of the antenna is costant):

1. *The arithmetic mean of the measurements (for the n number of measurements):*

- latitude: $$\varphi_{śr} = \frac{\sum_{i=1}^{n} \varphi_i}{n} \text{ [º] or [’]}, \quad (1.1)$$
- longitude: $$\lambda_{śr} = \frac{\sum_{i=1}^{n} \lambda_i}{n} \text{ [º] or [’]}. \quad (1.2)$$

2. *Deviations of measurements from the mean:*

For the measurement values and the arithmetic mean in minutes [’]:

- latitude: $$\Delta\varphi_i = (\varphi_i - \varphi_{śr}) \text{ [’] or [Nm]}, \quad (1.3)$$
- longitude $$a_i = (\lambda_i - \lambda_{śr}) \cdot \cos\varphi_{śr} \text{ [’] or [Nm]}. \quad (1.4)$$

3. *Sum of squares of deviations (square deviation of measurements):*

- latitude: $$\sum_{i=1}^{n} (\Delta\varphi_i)^2 \text{ [Nm}^2\text{]}, \quad (1.5)$$
- departure: $$\sum_{i=1}^{n} (a_i)^2 \text{ [Nm}^2\text{]}. \quad (1.6)$$

4. <u>*Mean square error (standard deviation of measurements):*</u>

Determined with a probability (or at the confidence level) of 0,683 is for:

- latitude: $$m_\varphi = \pm\sqrt{\frac{\sum_{i=1}^{n} (\Delta\varphi_i)^2}{n-1}} \text{ [Nm]}, \quad (1.7)$$
- departure: $$m_a = \pm\sqrt{\frac{\sum_{i=1}^{n} (a_i)^2}{n-1}} \text{ [Nm]}. \quad (1.8)$$

This means that, for a given parameter, the probability of an error occurring within, for example, the range from $-m_\varphi$ do m_φ is 0,683 %.

Respectivelyprobability, of an error occurrence within the following limits, equals::
-2m to 2m ... 0.955
-3m to 3m ... 0.997 (so-called maximum error defining the limit value for random errors)

5. An average error of the arithmetic mean:

- latitude : $$m'_{\varphi} = \pm \frac{m_{\varphi}}{\sqrt{n}} \text{ [Nm]}, \tag{1.9}$$
- departure: $$m'_{a} = \pm \frac{m_{a}}{\sqrt{n}} \text{ [Nm]}. \tag{1.10}$$

On the basis of the calculated mean square errors of the measurements φ and λ, a **root mean square error** called the average position error of the vessel may be determined:

$$M_0 = \sqrt{m_{\varphi}^2 + m_a^2} \text{ [Nm]} \tag{1.11}$$

The probability of the actual position within a **root mean square error (**its confidence level) is variable from 0.632 to 0.683, averaged 0.66%.

2. Determination of the mean position error of the hyperbolic coordinates

When using hyperbolic coordinates, i.e. when reading position line numbers (DECCA and LORAN-C systems), equations (2.1) to (2.5) should be used to calculate the mean square errors of the measurements:

$$l_{1śr} = \frac{\sum_{i=1}^{n} l_{1i}}{n}, \quad l_{2śr} = \frac{\sum_{i=1}^{n} l_{2i}}{n} \tag{2.1}$$

$$\sigma_1 = \pm \sqrt{\frac{\sum_{i=1}^{n} \left(l_{1i} - l_{1śr}\right)^2}{n-1}} \tag{2.2}$$

$$\sigma_2 = \pm \sqrt{\frac{\sum_{i=1}^{n} \left(l_{2i} - l_{2śr}\right)^2}{n-1}} \tag{2.3}$$

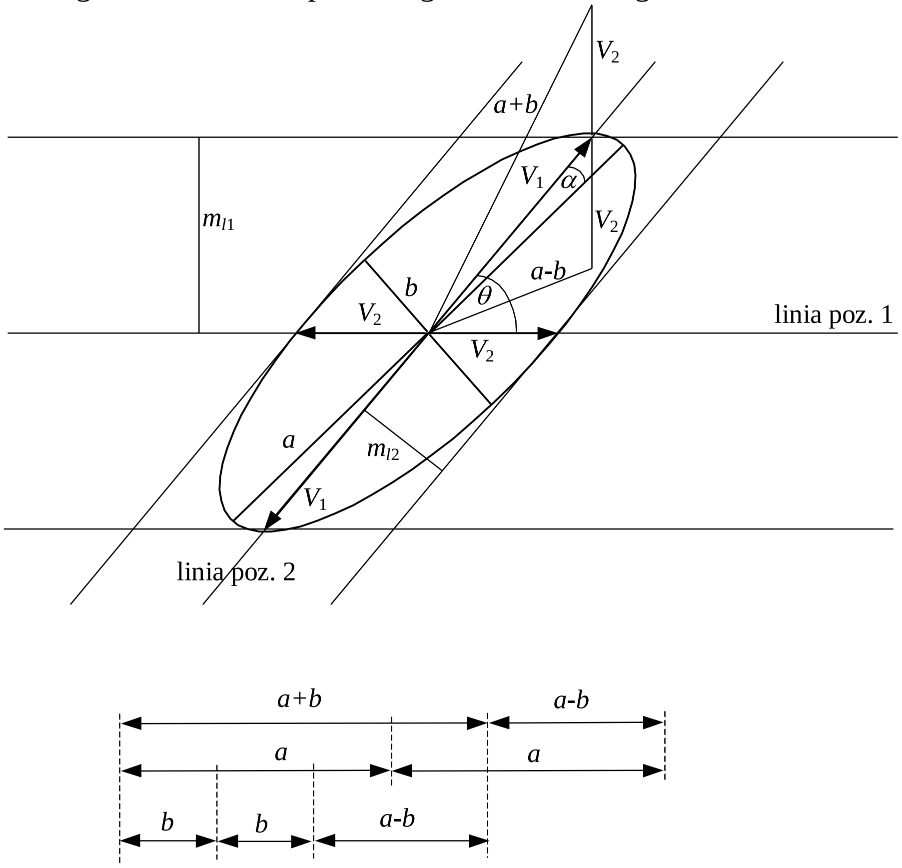
The values for Loran-C are in µs, for Decca in dimensionless units of phase difference. Correspondingly, the average square errors in the measurements in Nm are obtained by reading the width of the bands (w_l) of both hyperbolic nets from the map frame in Mm for the place (latitude) of observation for Decca and by calculating the amount of Mm per 1µ s for Loran-C:

- for Decca: $$m_{l1} = \sigma_1 \cdot w_{l1} \text{ [Nm]}, \; m_{l2} = \sigma_2 \cdot w_{l2} \text{ [Nm]} \tag{2.4}$$
- for Loran-C: $$m_{l1} = \sigma_1 \cdot w_{1\mu s} \text{ [Nm]}, \; m_{l2} = \sigma_2 \cdot w_{2\mu s} \text{ [Nm]} \tag{2.5}$$

The mean vector errors of the position lines can be determined by knowing the angle of θ, under which these lines intersect:

$$V_1 = \frac{m_{l1}}{\sin\theta}\ [\mathrm{Nm}],\ V_2 = \frac{m_{l2}}{\sin\theta}\ [\mathrm{Nm}] \qquad (2.6)$$

These errors are directed along the position lines: V_1 along the second position line, V_2 along the first position line creating a **parallelogram of errors** (probability of the position being inside the mean parallelogram of errors higher-axis 0,683×0,683=0,466).



Rys. 2.1. Geometric determination of mean parallelogram and mean error ellipse

The semiaxes of the mean ellipse of errors are as follows:

$$a = \frac{1}{2}\left(\sqrt{V_1^2 + 2V_1V_2\sin\theta + V_2^2} + \sqrt{V_1^2 - 2V_1V_2\sin\theta + V_2^2}\right) \text{ [Nm]} \tag{2.7}$$

$$b = \frac{1}{2}\left(\sqrt{V_1^2 + 2V_1V_2\sin\theta + V_2^2} - \sqrt{V_1^2 - 2V_1V_2\sin\theta + V_2^2}\right) \text{ [Nm]} \tag{2.8}$$

The angle α, between the semi-axis and the greater vector error is calculated from the formula:

$$\operatorname{tg} 2\alpha = \frac{\sin 2\theta}{\left(\frac{V_1}{V_2}\right)^2 + \cos 2\theta} \tag{2.9}$$

Angle α is placed away from the bigger vector error so that the semi axis is bigger and inside the angle θ formed by the arms V_1 and V_2.

The geometric method of determining the mean parallelogram of errors and the mean parallelogram of errors is shown in Fig. 2.1. The major semi axis of the ellipse is deposited on the two-century angle and . The perpendicular to it is the length of the minor semi axis b. The probability of finding the position of the vessel (laboratory) inside the mean ellipse of errors is smaller than the mean parallelogram of errors and is 0.393.

The average position error is determined by the relationship:

$$M_0 = \sqrt{V_1^2 + V_2^2} = \frac{1}{\sin\theta}\sqrt{m_{l1}^2 + m_{l2}^2} \text{ [Nm]} \tag{2.10}$$

3. Determination of the median and the modal random variable and the correlation coefficient of the two random variables.

Median and modal are among the positional measures of the surveyed statistical group.

For a convertible statistical population (for a random discrete variable), **median M*e* (second quartile)** divides the population into two equal parts; half of the units (of measurement) have values of characteristics less than or equal to the median, and half have values of characteristics greater than or equal to the M***e***. Therefore, the median is sometimes referred to as the **median's central value**.

In the detailed series, arranged in ascending order, the median is determined from the formula:

$$Me = \begin{cases} x_{\frac{n+1}{2}}, & gdy\ n\ jest\ nieparzyste \\ \frac{1}{2}\left(x_{\frac{n}{2}} + x_{\frac{n}{2}+1}\right), & gdy\ n\ jest\ parzyste \end{cases} \tag{3.1}$$

The position of the median shall be set at half the order of magnitude of the growing sample:

$$N_{Me} = \frac{n}{2} \tag{3.2}$$

Modal D (dominant, fashion, most common) is the value of a static characteristic that occurs most frequently in a given empirical distribution. Analytically for a calculable statistical collection (a calculable number of measurements) the modal share is determined from the formula:

$$D = x_{0d} + \frac{n_d - n_{d-1}}{(n_d - n_{d-1}) + (n_d - n_{d+1})} \cdot (x_{Gd} - x_{0d}) \qquad (3.3)$$

where: d - the number of the compartment(s) in which the modal split occurs,
x_{0d} - the lower limit of the compartment in which the modal split occurs,
x_{Gd} - górna granica przedziału, w którym występuje modalna,
n_d - size of the modal interval, i.e. the class with the number d,
n_{d-1}; n_{d+1} - the class sizes: preceding and following the modal interval

The correlation coefficient of two random variables x and y (Pearson) is a measure of the strength of a linear relationship between characteristics. It is calculated according to the relation:

$$r_{xy} = \frac{\sum_{i=1}^{n}(x_i - x_{śr})(y_i - y_{śr})}{\sqrt{\sum_{i=1}^{n}(x_i - x_{śr})^2 \cdot \sum_{i=1}^{n}(y_i - y_{śr})^2}} \qquad (3.4)$$

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